Experiment 6: Transient Behaviour Analysis

Abstract

The performance of first- and second-order systems in transient systems is explored in this experiment. The thermal and electrical forms of the first systems are studied. The transient response of a thermocouple is investigated in the thermal system, while the electrical system is investigated using a resistor-capacitor (RC) circuit. A single system, an inductor-resistor-capacitor (LRC) circuit, is examined for the second-order system. We found that time constant for covered is significantly more as compared to uncovered. In uncovered system, the heat transfer can easily take place time constant is less for temperature rise and decrees. The RC and LRC circuit shows consistency in their behaviour.

1 Introduction

The performance of first- and second-order systems in transient systems is explored in this experiment. The thermal and electrical forms of the first systems are studied. The transient response of a thermocouple is investigated in the thermal system, while the electrical system is investigated using a resistor-capacitor (RC) circuit. A single system, an inductor-resistor-capacitor (LRC) circuit, is examined for the second-order system.

To explore transient responses, all of the three uses a step input for excitation. The time constant, rising time, maximum overshoot percentage, peak time, and settling time are all time domain metrics that describe transient behaviour. The frequency bandwidth of a first-order system is calculated also.

2 Theory

Theories of all transient systems are discussed in the following sections.

2.1 First order thermal system of thermocouple

When constructing a measuring system, the dynamic response of a sensor is frequently taken into account. A first-order system can be used to model the response of a thermocouple temperature sensor. It will take some time for the thermocouple to respond to a sudden temperature change. The thermocouple will not be able to correctly depict the dynamic reaction to temperature changes if the response time is slow in relation to the rate of change of the temperature.

A basic heat transfer study is used to predict the response of a thermocouple. The pace where the sensors exchanges heat with its surroundings must be the same as the rate where the sensor's internal energy changes. If convection is the major mode of heat exchange (ignoring conduction and radiation), it would be for a thermocouple inside a fluid, the energy balance equation is:

$$hA(T_\infty - T) = mc\frac{dT}{dt}, \qquad (1)$$

After re-arranging the term, we can write it as,

$$\tau\frac{dT}{dt} + T = T_\infty \qquad (2)$$

Here **τ** is time constant, which is defined as,

$$\tau = \frac{mc}{hA} \ . \qquad (3)$$

Further, after solving the equation, we have

$$y(t) = kA\left(1 - e^{-t/\tau}\right). \qquad (4)$$

2.2 Second order electrical system of series LRC circuit

Series LRC circuit is governed by the second order different equation, which is shown below:

$$\frac{d^2 v_o}{dt^2} + \frac{R}{L}\frac{dv_o}{dt} + \frac{1}{LC} v_o(t) = \frac{1}{LC} v_s(t)$$

The solution of above differential equation helps to develop the equations for the frequency damping coefficient and other. A short description of each is given below:

Natural frequency:

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Damping ratio:

$$\zeta = \frac{\alpha}{\omega_0}$$

Logarithmic decrement:

$$\delta = \zeta \times \text{Td} \times \omega_0$$

3 Results

3.1 For covered thermocouple

For covered thermocouples, results of each trial is discussed in following points.

3.1.1 For trial 1

The time constant is determined by rate at which response is reached by 63.2 percent of final value.

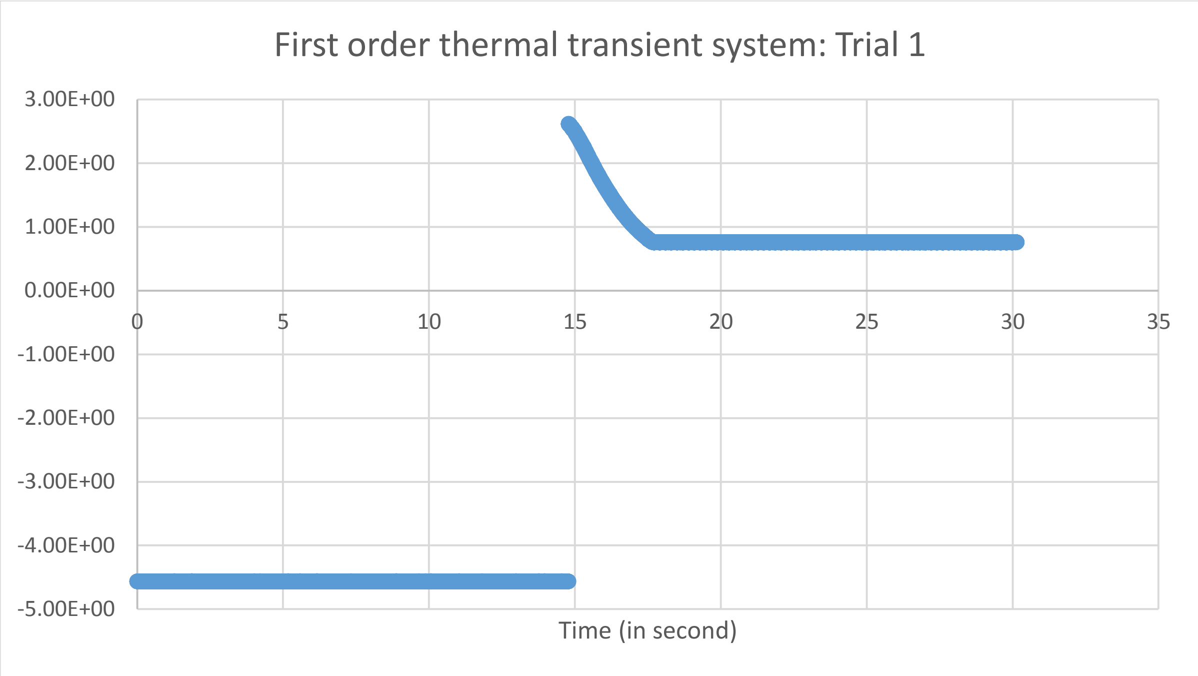

Figure 1 First order thermal transient system (For covered thermocouple): Trial 1

Taking at t = 0, step function is applied. Hence $y_0 = -4.57$

At $t = time\ constant\ (\tau)$ The response can be calculated by following equation.

We have,

$$\frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$y_\infty = 0.754$$

Hence,

$$y_t = y_\infty + (y_0 - y_\infty)e^{-1} = 0.3678y_0 + 0.6321y_\infty$$

After putting the values,

$$y_t = -1.204$$

Corresponding to this response time is, $\boldsymbol{\tau} =$ $\mathbf{15.000\ second}$ $(From\ excel\ table\ data.)$

$$Frequency\ bandwidth = \frac{1}{2\pi\tau} = \frac{0.159}{\tau} = \mathbf{0.0054\ hertz}$$

3.1.2 For trial 2

Taking at t = 0, step function is applied. Hence $y_0 = 2.72$

At $t = time\ constant\ (\tau)$ The response can be calculated by following equation.

We have,

$$\frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$y_\infty = 0.756$$

Hence,

$$y_t = y_\infty + (y_0 - y_\infty)e^{-1} = 0.3678y_0 + 0.6321y_\infty$$

After putting the values,

$$y_t = 1.478$$

Corresponding to this response time is, $\boldsymbol{\tau} = \mathbf{2.224\ second}\ (From\ excel\ table\ data.)$

$$Frequency\ bandwidth = \frac{1}{2\pi\tau} = \frac{0.159}{\tau} = \mathbf{0.0820\ hertz}$$

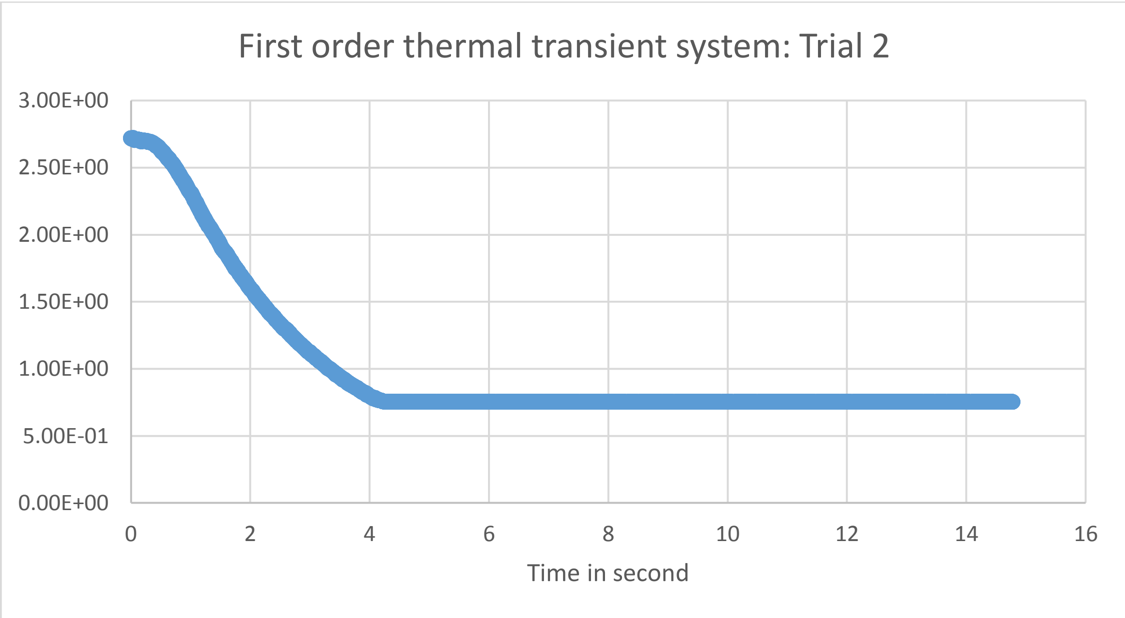

Figure 2 First order thermal transient system (For covered thermocouple): Trial 2

3.1.3 For trial 3

Taking at t = 0, step function is applied. Hence $y_0 = 2.94$

At $t = time\ constant\ (\tau)$ The response can be calculated by following equation.

We have,

$$\frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$y_\infty = 0.756$$

Hence,

$$y_t = y_\infty + (y_0 - y_\infty)e^{-1} = 0.3678y_0 + 0.6321y_\infty$$

After putting the values,

$$y_t = 1.559$$

Corresponding to this response time is, $\tau = 2.768\ \boldsymbol{second}\ (From\ excel\ table\ data.)$

$$Frequency\ bandwidth = \frac{1}{2\pi\tau} = \frac{0.159}{\tau} = \boldsymbol{0.0726\ hertz}$$

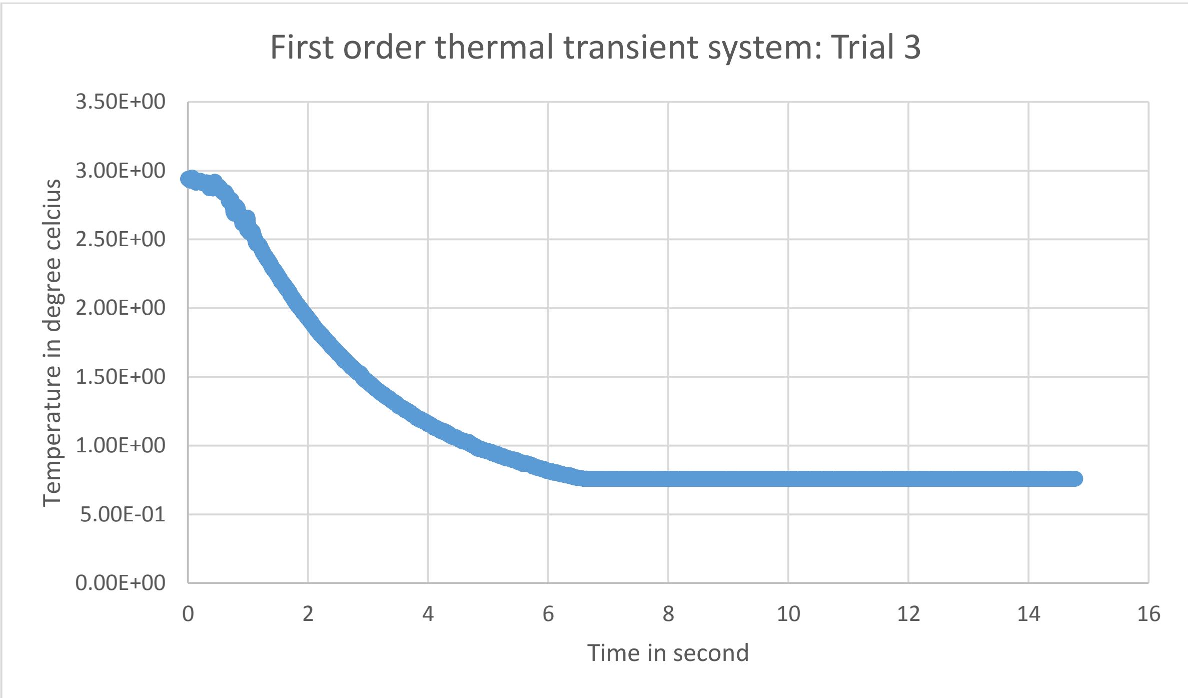

Figure 3 First order thermal transient system (For covered thermocouple): Trial 3

3.2 For uncovered thermocouple

For uncovered thermocouples, results of each trial is discussed in following points.

3.2.1 For trial 1

Taking at t = 0, step function is applied. Hence $y_0 = 2.90$

At $t = time\ constant\ (\tau)$ The response can be calculated by following equation.

We have,

$$\frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$y_\infty = 0.757$$

Hence,

$$y_t = y_\infty + (y_0 - y_\infty)e^{-1} = 0.367y_0 + 0.632y_\infty$$

After putting the values,

$$y_t = 1.543$$

Corresponding to this response time is, $\tau = 0.142\ \boldsymbol{second}\ (From\ excel\ table\ data.)$

$$Frequency\ bandwidth = \frac{1}{2\pi\tau} = \frac{0.159}{\tau} = \boldsymbol{1.120\ hertz}$$

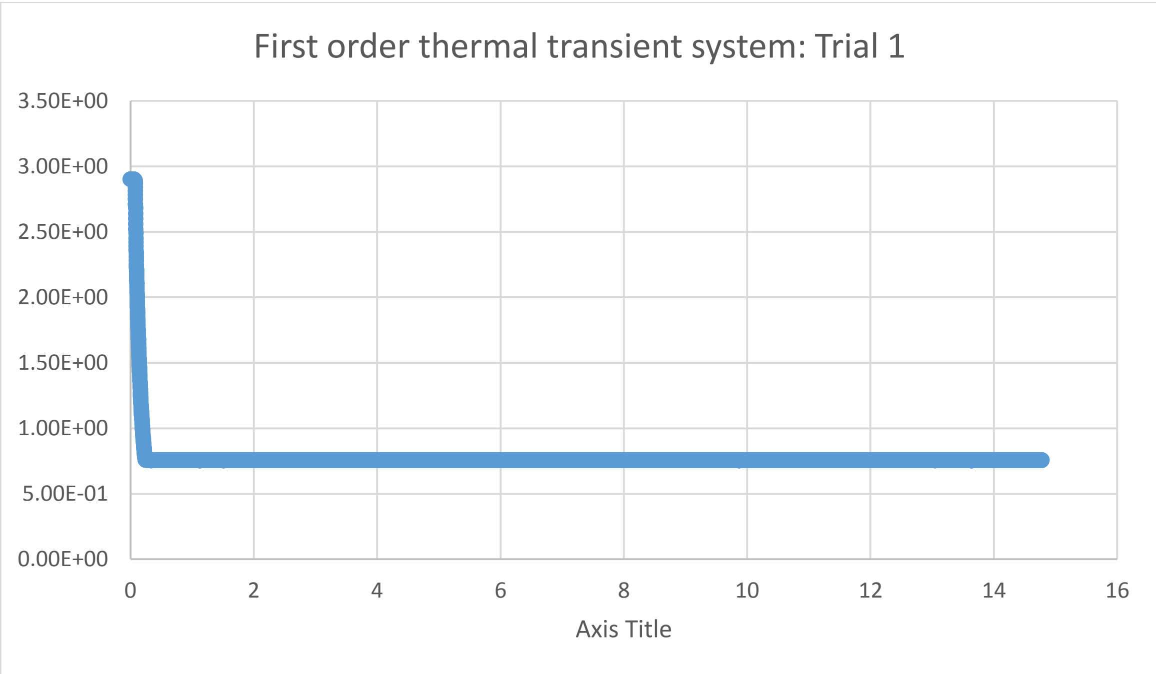

Figure 4 First order thermal transient system (For uncovered thermocouple): Trial 1

3.2.2 For trial 2

Taking at t = 0, step function is applied. Hence $y_0 = 2.81$

At $t = time\ constant\ (\tau)$ The response can be calculated by following equation.

We have,

$$\frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$y_\infty = 0.756$$

Hence,

$$y_t = y_\infty + (y_0 - y_\infty)e^{-1} = 0.3678y_0 + 0.6321y_\infty$$

After putting the values,

$$y_t = 1.511$$

Corresponding to this response time is, $\tau = 0.071$ $\boldsymbol{second}$ $(From\ excel\ table\ data.)$

$$Frequency\ bandwidth = \frac{1}{2\pi\tau} = \frac{0.159}{\tau} = \mathbf{0.2398\ hertz}$$

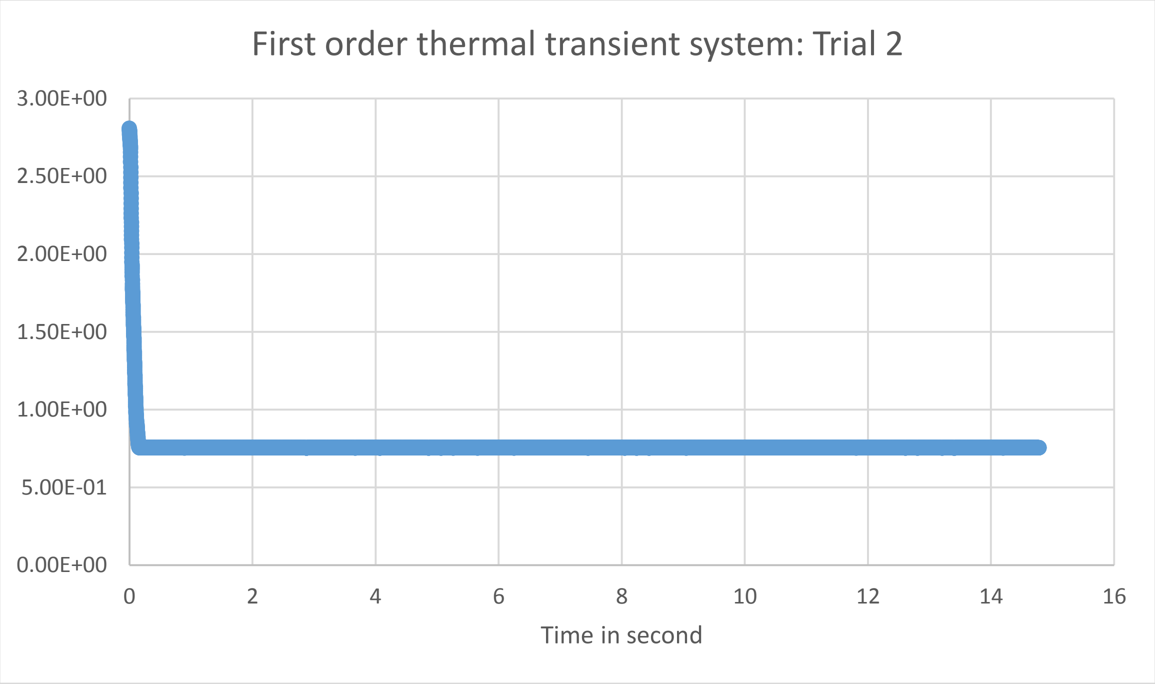

Figure 5 First order thermal transient system (For uncovered thermocouple): Trial 2

3.2.3 Trial 3

Taking at t = 0, step function is applied. Hence $y_0 = 3.14$

At $t = time\ constant\ (\tau)$ The response can be calculated by following equation.

We have,

$$\frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$y_\infty = 0.756$$

Hence,

$$y_t = y_\infty + (y_0 - y_\infty)e^{-1} = 0.3678y_0 + 0.6321y_\infty$$

After putting the values,

$$y_t = 1.633$$

Corresponding to this response time is, $\tau = 0.303$ $\boldsymbol{second}$ $(From\ excel\ table\ data.)$

$$Frequency\ bandwidth = \frac{1}{2\pi\tau} = \frac{0.159}{\tau} = \mathbf{0.524\ hertz}$$

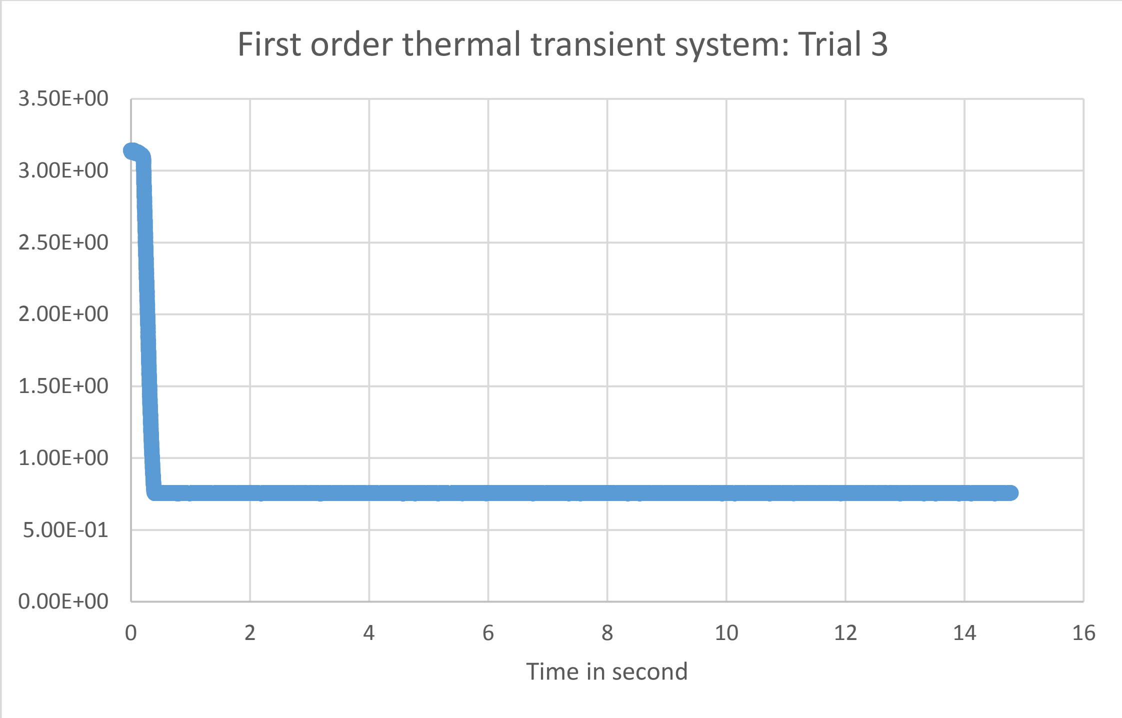

Figure 6 First order thermal transient system (For uncovered thermocouple): Trial 3

The calculated result is shown in the following table

Table 1 summarized result from thermocouple transient analysis

Trial	Condition	Initial Voltage (at room temp)	Time constant	Frequency bandwidth
1	Covered	-4.57	15.000	0.0106
2	Covered	2.72	2.224	0.0715
3	Covered	2.94	2.768	0.0574
		Mean	**6.664**	**0.047**
		Standard deviation	**5.899**	**0.026**
1	Uncovered	2.90	0.142	1.1197
2	Uncovered	2.81	0.071	2.2394
3	Uncovered	3.14	0.303	0.5248
		Mean	**0.172**	**1.295**
		Standard deviation	**0.097**	**0.711**

3.3 For RC circuit transient response system

3.3.1 Theoretical time constant

The formula to determine the time constant in RC circuits is:

$$\tau = RC$$

Resistor: 470 Ω ± 0.3%

Capacitor 3: ~6 µF ± 2%

Hence,

$$\tau = 470 \times 6 \times 10^{-6} = .002820$$

Uncertainty in time constant,

$$Uncertainty\ in\ time\ constant = 0.3 + 2 = 2.3\%$$

3.3.2 Time constant for first three steps

The response recorded from RC experiment in shown in following figure.

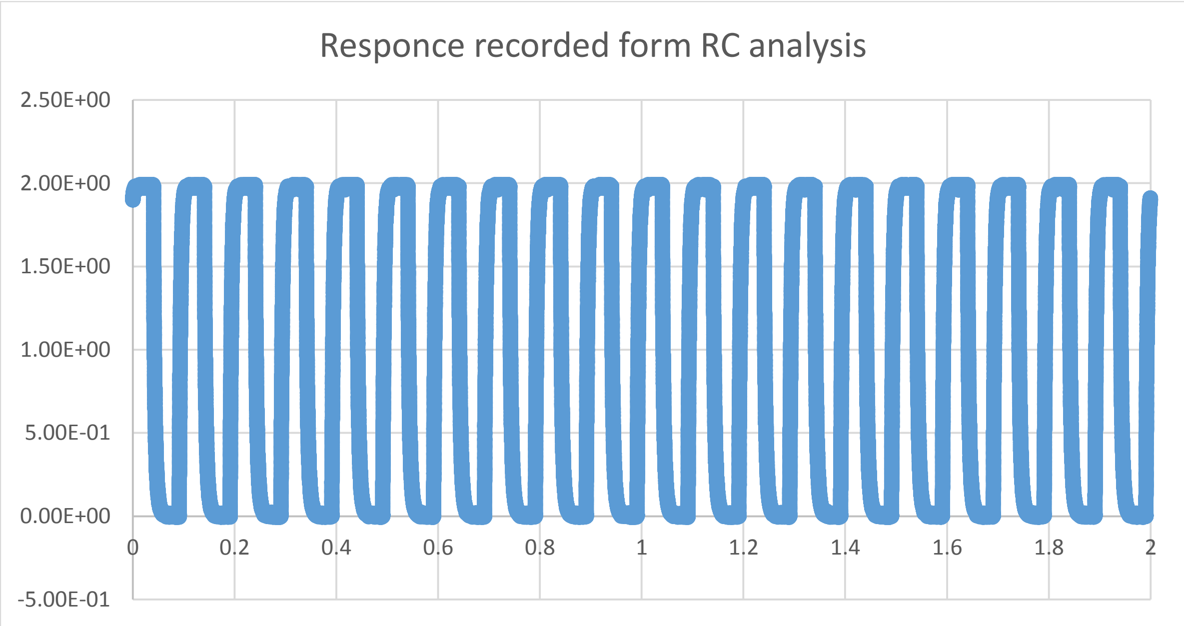

Figure 7 The response recorded from RC experiment

Considering the first step, (ignoring the falling edge)

Taking at t = 0, step function is applied. Hence $y_0 = 1.90$

At $t = time\ constant\ (\tau)$ The response can be calculated by following equation.

We have,

$$\frac{y_t - y_\infty}{y_0 - y_\infty} = e^{-t/\tau}$$

$$y_\infty = 1.98$$

Hence,

$$y_t = y_\infty + (y_0 - y_\infty)e^{-1} = 0.3678y_0 + 0.6321y_\infty$$

After putting the values,

$$y_t = 1.950$$

Corresponding to this response time is, $\tau = \mathbf{0.0024\ \ second}\ (From\ excel\ table\ data.)$

Similarly, in second step and third step response time is shown in following table,

Table 2 The time constant for each step

Step	Time constant
1	0.0024
2	0.0023
3	0.0028

3.4 For LRC circuit transient response system

3.4.1 Theoretical natural frequency

We have natural frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We have following values,

Resistor: 470 Ω ± 0.3%

Capacitor 1: ~0.06 µF ± 2%

Inductor: 470 mH ± 5%

After putting values, natural frequency:

$$Natural\ frequency, \omega_0 = \frac{1}{\sqrt{.470 \times .06 \times 10^{-6}}} = \mathbf{5955}\ \boldsymbol{rad/s}$$

Uncertainty in natural frequency:

$$Uncertainty\ in\ natural\ frequency = 0.5\ \times (5 + 2) = \mathbf{3.5}\%$$

3.4.2 Experimentally measured natural frequency

Data recorded from experiment is shown in the following chart.

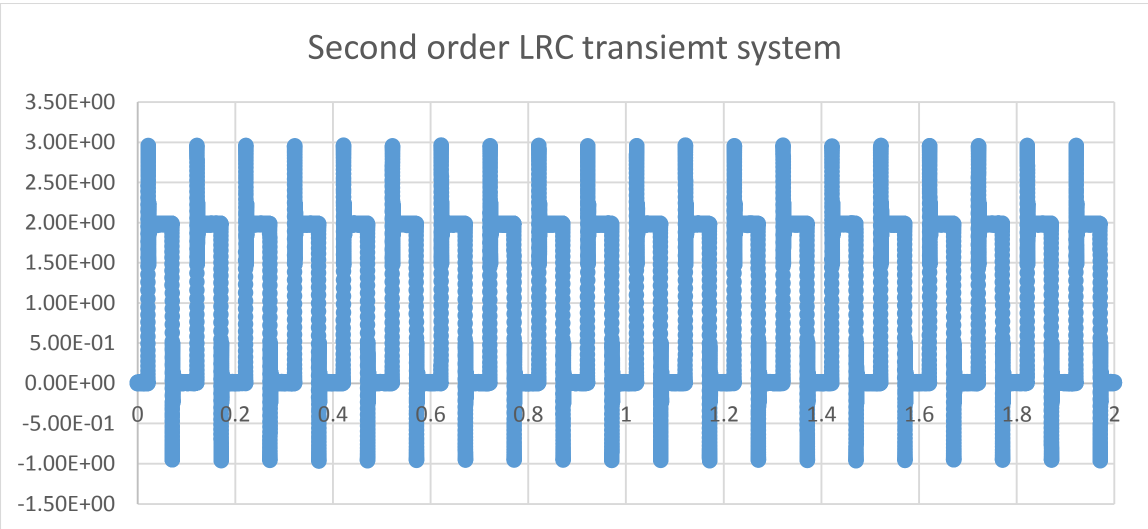

Figure 8 All step response (LRC circuit)

The first step from 0 to 0.1 second is shown in the following diagram

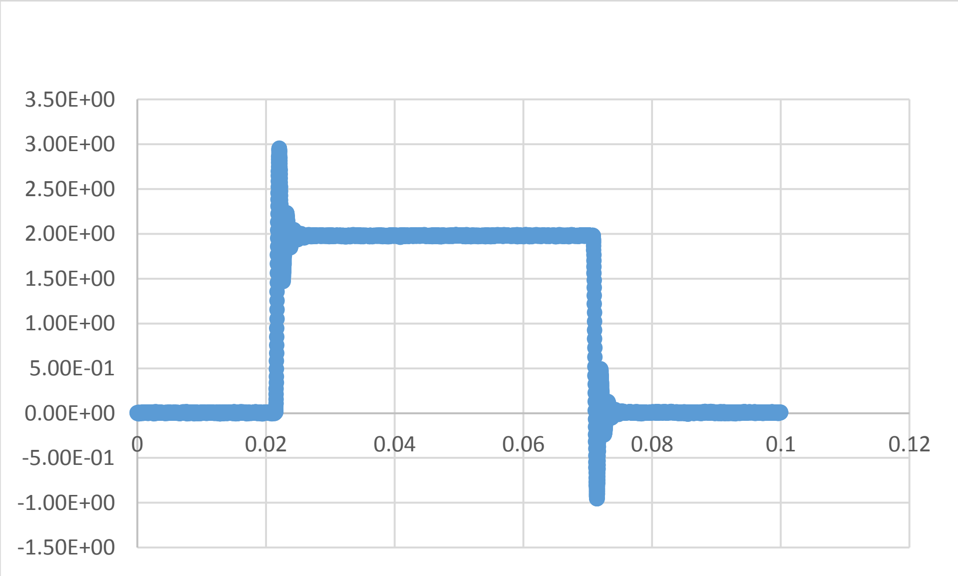

Figure 9 First step response (LRC circuit)

Using logarithmic decrement method,

$$\tau = \frac{t2 - t1}{\ln(\frac{V1 - v_f}{V2 - v_f})}$$

Taking v1=0.1V_f; v2=0.9V_f

$$V_f = 2.00$$

For, V1 = 0.2, t1 = 0.021588

For, V2 = 1.8, t2 = 0.021813

Hence, time constant is

$$\tau = \frac{0.000225}{\ln 9} = 0.000102402 \, second$$

Hence natural frequency,

$$Natural\ frequency, \omega_0 = \frac{1}{0.000102402} = 9765.42$$

Similarly the results obtained for step 2 and step 3 is shown in the following table,

Table 3 Natural frequency (rad/s), Td, δ and ζ

Step	Natural frequency (rad/s)	Td	δ	ζ
1	9765.42	0.000643	321.543	51.201
2	8755.24	0.000717	358.642	57.109
3	9428.33	0.000666	333.039	53.032
Theoretical	5955	0.001055	527.288	83.963

3.4.3 Peak time, peak overshoot (OS%), rise time, and settling time ($T_{s,1\%}$)

Table 4 Peak time, peak overshoot (OS%), rise time, and settling time (Ts,1%)

Step	Natural frequency (rad/s)	Peak time	Rise time	Setting time
1	9765.42	0.04330872	0.0000044	0.0000092
2	8755.24	0.043303634	0.0000044	0.0000092
3	9428.33	0.043306961	0.0000044	0.0000092
Theoretical	5955	0.043292437	0.0000044	0.0000092

4 Discussion

4.1 Part A: Thermal System Transient Response

We discussed the thermal system transient response of thermocouple. Thermocouple is exposed to a step temperature source in this experiment. To determine the system's time constant, the response is measured and the data is altered. The calculated value in shown in the Table 1. The experiment was performed in covered and uncovered setup. We found that time constant for covered is significantly more as compared to uncovered. In uncovered system, the heat transfer can easily take place time constant is less for temperature rise and decrees.

4.2 Part B: Electrical System Transient Response

The results of electrical system transient response is shown in the table 2, 3 and 4. The time constant for each step of RC system is approximately same. The LRC system shows the overdamped behaviour. A response that is overdamped does not fluctuate around the steady-state value but requires longer achieving steady-state than a response that is severely damped. In a static or memoryless system, there is no transient reaction. The output does not always match the input in dynamical system or systems containing memory components like capacitors and inductors. They also display some response that is solely reliant on the present condition of the memory and storage elements, the system's order, and the type of input applied. As a result, while reaching the steady state, the output waveform has peaks and oscillations.

5 Conclusion

The transient response of a thermocouple is investigated in the thermal system, while the electrical system is investigated using a resistor-capacitor (RC) circuit. A single system, an inductor-resistor-capacitor (LRC) circuit, is examined for the second-order system. We found that time constant for covered is significantly more as compared to uncovered. In uncovered system, the heat transfer can easily take place time constant is less for temperature rise and decrees. The RC and LRC circuit shows consistency in their behaviour.

6 Reference

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